RESOLVING POWER

SYNOPSIS: In this lab you will measure the resolving power of your eyes and of binoculars. You will determine the theoretical diffraction limit for each, and compare with your observed resolving power.

EQUIPMENT National Bureau of Standards resolution charts, tape measure, binoculars.

REVIEW: Degrees and Radians, Small Angle Approximation (pages 19-23)

Resolving power is the ability of an instrument or observer to separate two very close images. Think of two persons walking away from you along a very long straight street. You will initially have no difficulty in seeing the two persons clearly. As they get farther and farther away there will come a time when you can no longer see that there are two persons - their images will have merged. When you just see the images merging, you have reached the limit of resolution of your eyes.

Similarly, when looking at a pair of stars in a telescope, you can see two separate images if the stars are far enough apart. You cannot see distinct images if the stars are too close together.

Resolving power is an angle; specifically, it is the angle subtended at your eye by the two objects when their images just start to merge. For example, in the diagram below s is the actual distance between the stars, and r is the actual distance from the observer to the stars. Therefore, if \( q \) is in degrees,

\[
\tan (q) = \frac{s}{r} = \frac{\text{opposite}}{\text{adjacent}}
\]

However, by using the small angle approximation, the expression for resolving power is simpler when we express the angle \( q \) in radians:

\[
\theta = \frac{s}{r}
\]
\[
q = \{ \text{EQ} \, f(s,r) \} \text{radians}. \quad (1)
\]

Note that both \(s\) and \(r\) must have the same units (e.g., both in mm or both in meters, or whatever) so that the units will cancel and yield the (dimensionless) angle \(q\) expressed in radians. It is convenient to convert from radians to minutes of arc or seconds of arc:

\[
1 \text{ radian} = 57.3 \text{ degrees} = 3,438 \text{ arc minutes} = 206,265 \text{ arc seconds}
\]

Part I. Measuring The Resolving Power of the Eye and Binoculars

To measure resolving power, you will use wall charts (from the National Bureau of Standards) with a series of closely spaced black lines on a white background. You will pick a pair of lines and step backwards from the chart until that pair has merged. Then \(s\) is the distance from the center of one black line to the center of the next, and \(r\) is the distance from the wall when you can just barely tell the lines apart. Equation (1) is used to calculate the observed resolving power, which we will call \(q_{\text{observed}}\).

\[
\frac{s}{r} = \{ \text{EQ} \, f(s,r) \} \text{radians}
\]

With these charts, you do not need actually to measure \(s\). Just note \(N\), the number next to the group of lines you have chosen. The resolving power is then

\[
\begin{align*}
\text{Chart A:} & \quad s = \left(\frac{70}{N}\right) \text{ mm} \\
\text{Chart B:} & \quad s = \left(\frac{11}{N}\right) \text{ mm}
\end{align*}
\]

1.1 First determine the resolving power of each of your eyes separately. (Wear your usual glasses or contact lenses.) Illuminate the charts with a lamp, and stretch out the tape measure from the wall backwards. Make a note of which chart you are using (A or B). Pick a set of lines and make a note of the value of \(N\) for the set. Walk backwards until the lines appear to merge together. Walk forwards and backwards a few feet to be sure that you really do have that group of lines at the limit of resolution. Measure your distance from the chart. Repeat a couple of times, using different sets of lines on the different trials.
1.2  Find \(s\) (as described above) and calculate \(q_{\text{observed}}\). First make sure that you have \(s\) and \(r\) in the same units, so that you will get \(q_{\text{observed}}\) in radians; then convert to arc minutes. Average your results for each eye separately. Do your eyes have about the same resolving power? Or is one eye appreciably better than the other (this sometimes happens)? Then average your two eyes together; call this \(q_{\text{observed}}\) for your eyes.

1.3  The full Moon appears about 1/2 of a degree, or 30 arc minutes, in diameter. (Remember: the angular size of the Moon is about the same size as a fingernail at arm's length.) On its face, typical maria (the dark patches that make up the features of "the man in the moon" are about 5 arc minutes across, while large craters are about 0.8 arc minutes across. Using the value of \(q_{\text{observed}}\) for your eyes, would you expect to be able to resolve (identify the presence of) maria? Would you expect to see individual craters with your naked eye? How do these predictions compare with what you have actually observed on the Moon with your naked eye?

1.4  Find the resolving power \(q_{\text{observed}}\) of the binoculars in the same manner; you need not do this one eye at a time. You will probably need to use chart B. Of course, you are really finding the combined resolving power of the binoculars and your eyes.

1.5  Compare \(q_{\text{observed}}\) for the binoculars with \(q_{\text{observed}}\) for your eyes. Which is larger? Why? Is it "better" for \(q_{\text{observed}}\) to be large or to be small?

1.6  Using binoculars, would you expect to be able to resolve maria and craters on the Moon? Why or why not?

Part II. The Diffraction Limit

As a beam of light passes the edge of an object along its path (such as the edge of a mirror or lens), the beam becomes slightly spread out; this process is called diffraction. The phenomenon of diffraction limits the sharpness that can be achieved in the images formed by a telescope.

An important result from the theory of diffraction is that the resolving power of an instrument with a circular aperture of diameter \(d\) is limited to an angle

\[
q_{\text{diffraction}} = \{ EQ \over F(1.2 \times \text{Wavelength},\text{Diameter}) \} = \{ EQ \over F(1.2 \ \lambda,d) \}
\]
where \( l \) is the wavelength of the light being used. For visible light, you may take \( l = 5.5 \times 10^{-4} \text{ mm} \). (The subscript \( \text{diffraction} \) stands for "diffraction limit.")

Note that diffraction limit does not guarantee how good an optical system actually is; it merely states how good the system theoretically could be if everything were "perfect". If the incoming light is blurred for some reason, if the optics were flawed, or if the optical "detector" can't make full use of the fine detail produced by the lens or mirror, then the actual resolution limit will be worse (larger) than the theoretical limit.

II.1 Calculate \( q_{\text{diffraction}} \) for your eyes, assuming that the diameter of your eye pupil is 3 mm. Convert from radians to arc-minutes. Compare to \( q_{\text{observed}} \) (that is, to the final average for your eyes).

II.2 Speculate on possible reasons why the actual resolving power \( q_{\text{observed}} \) that you measured is not as good as the theoretical resolving power \( q_{\text{diffraction}} \).

II.3 Many predatory animals, such as eagles, are known for their keen eyesight, yet their eyes are subject to the same diffraction limitation that human eyes are. List several ways their "optical systems" might be better adapted than ours so as to give them superior vision for hunting.

II.4 Calculate \( q_{\text{diffraction}} \) for the binoculars. The diameter of the binocular lens is probably 50 mm, but you might want to verify this: binoculars are classified by the expression

\[
(\text{magnification}) \times (\text{diameter of lens in mm})
\]

For example, a pair of 7 x 50 binoculars means that it magnifies the angle subtended by the object by seven times, using 50 mm diameter lenses.

II.5 Compare \( q_{\text{diffraction}} \) for the binoculars to \( q_{\text{observed}} \) for the binoculars. Again, which is larger and why?

II.6 What is the theoretical diffraction limit for the 200 inch (5 meter) diameter telescope on Mt. Palomar?

II.7 In actual practice, the telescope on top of Palomar Mountain can rarely resolve stars closer together than one arc-second. (The parallax shift in stellar positions due to the Earth's motion around the Sun, which is our principle means of measuring their distances, is smaller than one arc-second even for the nearest stars! Hence astronomers must resort to exotic methods to resolve such apparent motions.) On the other hand, the Hubble Space Telescope, which is
smaller in diameter and has (unfortunately) flawed optics, is still able to resolve objects several times smaller than an arc-second. What do you think is the reason? (Hint: the answer has to do with the locations of the telescopes, not with their size or optical quality).